PARTIAL DIFFERENTIAL EQUATIONS

MATH 8250

Course Description:

Partial differential equations (PDEs) are fundamental in the application of mathematics to science and engineering. Topics to be covered will include: Linear and nonlinear first-order equations, classification of second-order linear equations, elliptic, hyperbolic and parabolic equations and boundary value problems, and Green's functions. **3 credits**

Prerequisites:

MATH 1970, MATH 2350, or instructor's permission. MATH 4330/8336 is recommended, but not required.

Overview of Content and Purpose of the Course:

This course introduces the student to PDEs, their theoretical foundations, and their applications, which include optics, propagation of waves (light and sound), electric field theory, diffusion, and fluid dynamics. PDEs are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to PDEs. This course covers the basic methods of PDEs. The main goal is to provide a solid conceptual understanding of some of the basic topics in the field of PDEs. The students will be exposed to both theoretical and applied points of view. Many different techniques for solving these equations are discussed. Standard topics such as linear and nonlinear first-order equations, classification of second-order linear equations, elliptic, hyperbolic and parabolic equations and boundary value problems, Green's functions, and Fourier series are included.

This is a graduate level course in PDEs and their applications in mathematical physics. It is designed to meet the needs of science, engineering, physics, and mathematics students. We will cover basic techniques for analyzing PDEs and focus on several particular types of PDEs (linear and nonlinear) that allow us to find explicit solution formulas. The majority of the topics covered in this course are theoretical. We will discuss many techniques, such as the method of characteristics, separation of variables, Fourier transform, and many others. The material presented is applicable to any field of study that makes use of PDEs to model its phenomena, whether that field is physics, finance, electrical engineering, or anything else.

This course covers the following materials: Transport equations; Characteristics; Classification of PDEs; Boundary value problems; Wave equations Heat/diffusion equations; Laplace equations; Maximum principle; Separation of variables; Fourier series; Distributions; Green's functions.

Anticipated Audience/Demand:

Graduate students in Mathematics, Engineering, and Computer Science needing the basic Mathematical theory and applications of Partial Differential Equations.

Major Topics:

1) First-order equations by the method of characteristics; linear, quasilinear, and nonlinear equations; applications to traffic flow and geometrical optics.

2) Principles for higher-order equations; power series and Cauchy Kowalevski theorem; classification of second-order equations; linear equations and generalized solutions.

3) Wave equations in various space dimensions; domain of dependence and range of influence; Huygens' principle; conservation of energy, dispersion, and dissipation; applications to light and sound.

4) Laplace's equation; mean values and the maximum principle; the fundamental solution, Green's functions, and Poisson kernels; properties of harmonic functions; applications to vector fields.

5) The heat equation; eigenfunction expansions; the maximum principle; Fourier transform and the Gaussian kernel; regularity of solutions; scale invariance and the similarity method; applications to fluid dynamics.

Methods:

The class will be presented primarily in lecture form with student discussion encouraged. Questions are encouraged in class and out.

Student Role:

Students must attend and participate in class in addition to completing course requirements. Students are expected to do reading and assignments as they are assigned.

Textbook:

Hattori, Harumi. *Partial Differential Equations: Methods, Applications and Theories*. Singapore: World Scientific Publishing Company, 2013.

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